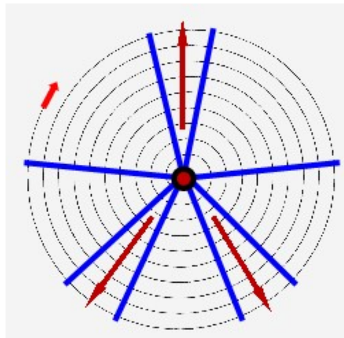
The Prime Spiral Sieve

**Radial Geometry and Chordal Algorithms
Demystify the Prime Number Sequence**



"My general view of mathematics is that most of the complicated things we learn have their origins in very simple examples and phenomena." – Dr. Richard Evan Schwartz, Chancellor's Professor of Mathematics, Brown University

"Deep and satisfying mathematics can arise from simple beginnings." – Vicky Neale, PhD, Additive Number Theory

"Everything should be made as simple as possible, but not simpler." – Albert Einstein

"Simplicity is the ultimate sophistication." – Leonardo Da Vinci

"Seek simplicity and distrust it." – Alfred North Whitehead

Introduction

This site explores a deterministic algorithm and geometry in the form of a spiral sieve encompassing eight factorization progressions that intertwine like an octal helix and ultimately determine the distribution of prime numbers greater than five (5). (Prime numbers defined as natural numbers whose only factors are 1 and themselves.)

To date this site has attracted more than 79,600 "absolute unique visitors" (Google Analytics speak) from 9,072 cities representing every country on Earth. Scores have returned repeatedly, and not one has offered counter-examples. Commenting on LinkedIn.com, a NASA data scientist said of this work: It "is truly a well thought out labor of mathematical thought." From a hermeneutic perspecitve, the patterns explored herein speak eloquently for themselves ... *Res ipsa loquitur ...*

The sequence populating the Prime Spiral Sieve (pictured below) can be variously defined as:

♦ Natural numbers not divisible by 2, 3 or 5. And given no prime number > 5 is divisible by 2, 3 or 5, it's axiomatic that the domain contains all prime numbers > 5, starting with 7 (the number 1 being "silent")... and their multiplicative multiples, beginning with 7 x 7 = 49, the first composite number in the sequence. It follows that all members of our domain are relatively prime (aka coprime or mutually prime) to 2, 3 and 5.

- ♦ Natural numbers ≡ {1, 7, 11, 13, 17, 19, 23, 29} modulo 30, which parse into 8 arithmetic progressions, each with a common difference of 30 between consecutive terms.
- ♦ 1 {+6 + 4 + 2 + 4 + 2 + 4 + 6 + 2} {repeat ... ∞}.
- ♦ 30n+1, 30n+7, 30n+11, 30n+13, 30n+17, 30n+19, 30n+23, 30n+29.
- ♦ Natural numbers modulo 30 that distribute to the following 8 angles: 12° ... 84° ... 132° ... 156° ... 204° ... 228° ... 276° ... 348°.
- ♦ All odd numbers with digital root of 1, 2, 4, 5, 7 or 8 and final digit of 1, 3, 7 or 9 ... the first 8 of which are 1, 7, 11, 13, 17, 19, 23, 29.

It's important to note that the beautiful symmetries encountered within this domain, both numeric and geometric, are largely a consequence of patterns rooted in its period-24 digital root, which has the following repetition cycle: {1, 7, 2, 4, 8, 1, 5, 2, 4, 1, 5, 7, 2, 4, 8, 5, 7, 4, 8, 1, 5, 7, 2, 8} {repeat ...}. This is equivalent to three rotations around the Prime Spiral Sieve.The first 24 members of our domain are thus especially important in relation to these cycles. For convenience, and to emphasize this modulo 90 periodicity, we occasionally frame them as: Numbers ≡ to {1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59, 61, 67, 71, 73, 77, 79, 83, 89} modulo 90.

The modulo 90, period-24 digital root repetition cycle of our domain allows us to sieve and sequence by digital root (dr), as listed below (and documented by the author on the *On-Line Encyclopedia of Integer Sequences*):

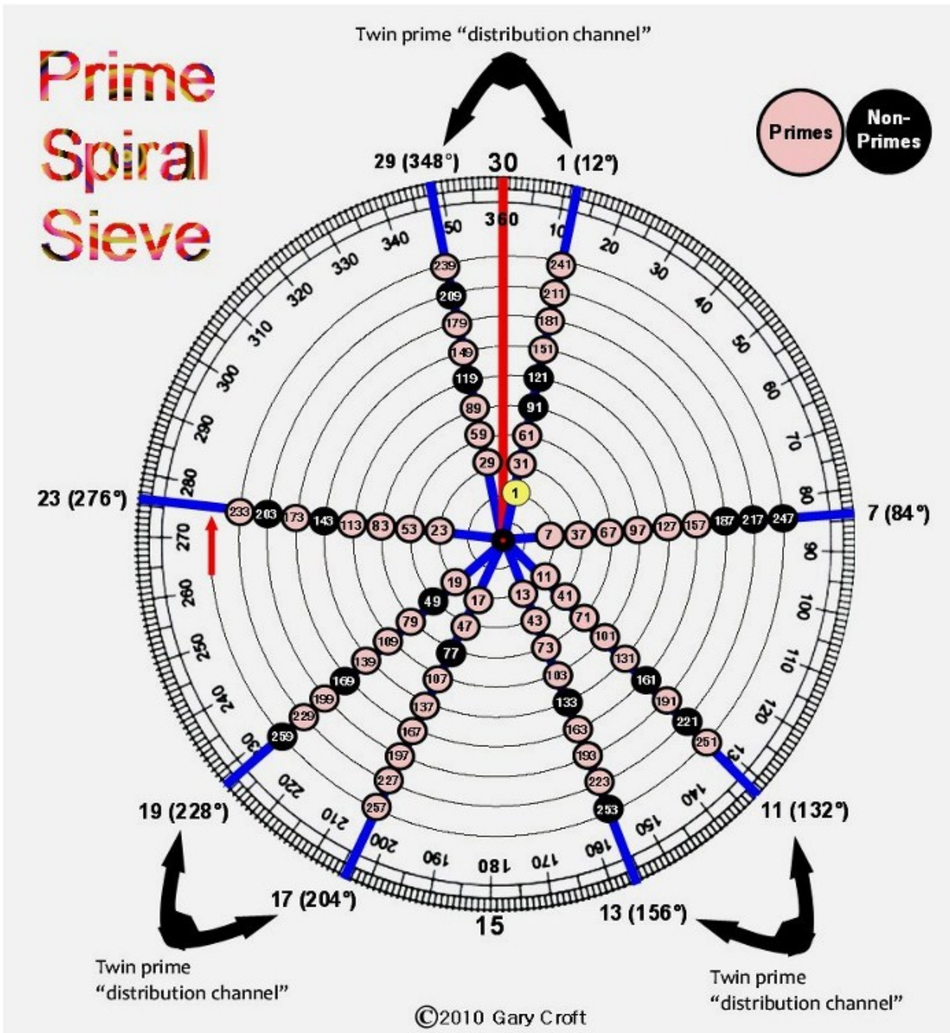
dr 1 = n ≡ {1, 19, 37, 73} mod 90: 1{+18+18+36+18} {repeat ...} (oeis.org/A301617)
dr 2 = n ≡ {11, 29, 47, 83} mod 90: 11{+18+18+36+18} {repeat ...} (oeis.org/A301621)
dr 4 = n ≡ {13, 31, 49, 67} mod 90: 13{+18+18+18+36} {repeat ...} (oeis.org/A301622)
dr 5 = n ≡ {23, 41, 59, 77} mod 90: 23{+18+18+18+36} {repeat ...} (oeis.org/A301623)
dr 7 = n ≡ {7, 43, 61, 79} mod 90: 7{+36+18+18+18} {repeat ...} (oeis.org/A301628)
dr 8 = n ≡ {17, 53, 71, 89} mod 90: 17{+36+18+18+18} {repeat ...} (oeis.org/A295869)

When the modulo 90 matrix listed above is color-coded to highlight its 12 lateral 90-sums (12 x 90 = 1080 = 360 x 3), perfect symmetry is exposed:

In our section devoted to twin primes we demonstrate how to configure the above table to sieve and/or sequence twin prime candidates by digital root dyads.

[Note: The easiest and fastest way to determine modulo 90 congruence of a whole number is to reduce all digits antecedant to the number's terminating digit to their digital root. For example, the 1st three digits of 2497 reduce to digital root 6 leaving 67 which equates to 2497 ≡ {67} mod 90. Keep in mind that 9's count as zero when calculating digital root. This is sometimes called 'casting out 9's.']

Below is a picture of the Prime Spiral Sieve showing rotations spanning numbers 1 thru 259 of the infinite sequence populating the sieve, as defined above:



Hidden deep within this sieve, a radical spin on modulo 30 wheel factorization, are mysteriously beautiful symmetries and geometries, profound in their implications. This sieve not only forms the basis of an extremely efficient and widely used prime number sieving algorithm of the 'non-probabilistic' kind, but it demonstrates how prime numbers are distributed within a radial geometry that effectively defragments the Ulam Spiral and ultimately leads us to "the theory of everything."

This domain, once fathomed, reveals itself to be a beautiful mathematical object. It can be conceived as both an infinite spiral and–when matrix factorized at the digital root level–an ever-expanding 4-sided pyramid structured at the deepest level by palindromic sequences. Regardless, the real power of this geometry becomes evident when we triple it dimensionally and explore modulo 90 factorization symmetries at the digital root level, culminating in the Magic Mirror Matrix, a 'calculatory geometry' that serves as a prime factorization sequencer accounting for the first 1000 prime numbers, *exactly*, and ultimately all prime numbers > 5.

Most profoundly, we'll discover how the eight spiraling factorization algorithms, i.e., the multiplicative multiples of the domain's sequence starting with 7^2 that ultimately account for *every* composite number within the Prime Spiral Sieve's orbits, are structured by rotational symmetry groups in the shape of equilateral triangles ({1,4,7} {2,5,8} {3,6,9}) that form 3 x 3 matrices which in turn extrapolate into beautiful complex polygons.

Only elementary arithmetic and geometry are required to understand what you find on this site. This minimal requirement is reflected in a quote from *Prime Numbers and the Riemann Hypothesis* by Mazur and Stein: "It is striking how little you actually have to know in order to appreciate the revelations offered by numerical exploration."

Eighteen of the sequences discussed on this site have been published by the author on the *On-Line Encyclopedia of Integer Sequences*. Here's a link to them: OEIS Listings for Gary W. Croft

Also, an M.I.T. licensed (Python) Prime Factorization Tool compared seven non-probabilistic prime number sieving algorithms, and the programmer deemed the Croft Spiral (aka Prime Spiral Sieve) the 'fastest and most efficient' of those tested. Quoting the programmer, "The fastest method, Croft, is over 1000 times faster than the slowest."

Further validation comes from Fredrick Michael, PhD (Director Experfy Institute / Harvard Innovation Laboratories), who utilized this site's period-24 model of primes and composites in a study of statistics and modular arithmetic of prime numbers. An abstract of the referenced study can be found at Notes on Prime Numbers, Their Numerical Statistics & Patterns I: Modular Arithmetic and the Eight Fold Period 24 Model. The period-24 patterns in question, relating to modulo 90 factorization algorithms and the digital root dyad cycles generated by twin primes, are discussed at length on our page devoted to demystifying Twin Primes.

And here's a link to a .pdf version of Croft's paper titiled "From Vedic Square to the Digital Root Clockworks of Modulo 90 Factorization" published in the September, 2018 edition of the *Journal of Vedic Mathematics.*

Foundations

The genesis of most if not all repeating prime number patterns described in the mathematics literature, e.g., Twin Primes, Cullen Primes, Chen Primes, Sexy Primes, Cousin Primes, Sophie Germain Primes, Siamese Primes, Cunningham Chains ... the list goes on and on ... can be readily deciphered using the Prime Spiral Sieve as an analytical tool employing modular arithmetic (and specifically, modulo 30 relationships). Here are two examples supporting this claim, i.e. using this sieve to analyze and predict Siamese Primes (n^2 -2 and n^2 +2 are primes) and Sophie Germain Primes (p and 2p+1 are primes), keeping in mind that these will make more sense after you've read what follows.

The most obvious of these repeating prime patterns are the three Twin Prime Distribution Channels, described at length on this site. These and all other such repeating–albeit intermittent and seemingly random–patterns are fundamentally sub-patterns of the set of natural numbers not divisible by 2, 3 or 5 when arrayed in 8 dimensions, whether in a matrix or spiral form. The illusion of randomness results from the overlapping sequences of the eight algorithmic "chord progressions" that factorize the domain. We will be discussing these progressions when we get to prime factorization.

The first rotation of the sieve, comprised of 8 members, (1, 7, 11, 13, 17, 19, 23 and 29), is the deterministic key to everything that follows. These are the first 8 counting numbers not divisible by 2, 3 or 5, a sequence which by definition includes (and *only* includes) 1 and all primes ≥ 7 and their multiplicative multiples (and, as you'll see below, it's conjectured that the entire set can be generated by a simple expression involving 2, 3 and 5). The inference of this conjecture is that *all* prime numbers greater than 5, i.e., starting with 7, can be produced by this expression.

[Note: Given our domain is limited to numbers ≡ {1,7,11,13,17,19,23,29} modulo 30," only ϕ(m)/m = 8/30 or 26.66% of natural numbers need be sieved. Also note that if you plug the number 30 into Euler's totient function, phi(n): phi(30)= 8, with the 8 integers (known as totatives) smaller than and having no factors in common with 30 being: 1, 7, 11, 13, 17, 19, 23 and 29, i.e., what are called "prime roots" above. Thirty is the largest integer with this property.]

The integer **30**, product of the first three prime numbers (2, 3 and 5), and thus a primorial, plays a powerful role organizing the array's perfect symmetry, viz., in the case of the 8 prime roots:

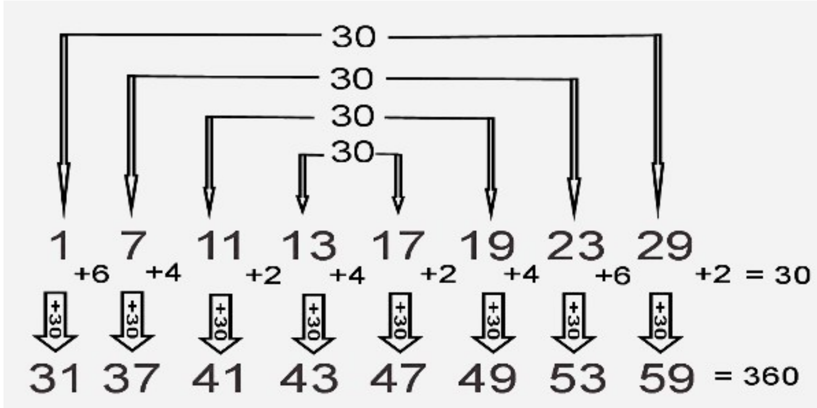
1+29=30; 7+23=30; 11+19=30; and **13+17=30.**

In *The Number Mysteries* well-known physicist and mathematics popularizer Marcus Du Sautoy writes: "In the world of mathematics, the numbers 2, 3, and 5 are like hydrogen, helium, and lithium. That's what makes them the most important numbers in mathematics." Although 2, 3 and 5 are the only prime numbers not included in the domain under discussion, they are nonetheless integral to it: First of all, they sieve out roughly 3/4ths of all natural numbers, leaving only those nominally necessary to construct a geometry within which prime numbers can be optimally arrayed. The remaining 26.66% (to be a bit more precise) constituting the array can be constructed with an elegantly simple interchangeable expression that incorporates the first three primes. It's conjectured that this expression can be configured (albeit by trial-and-error) to produce **all** (*and only*) the numbers in the array (and their negatives): $\mathbf{x^n y^n \pm z^n}$ where x=2, y=3 and z=5. Thus: $\mathbf{x^n z^n \pm y^n}$ and $\mathbf{y^n z^n \pm x^n}$.

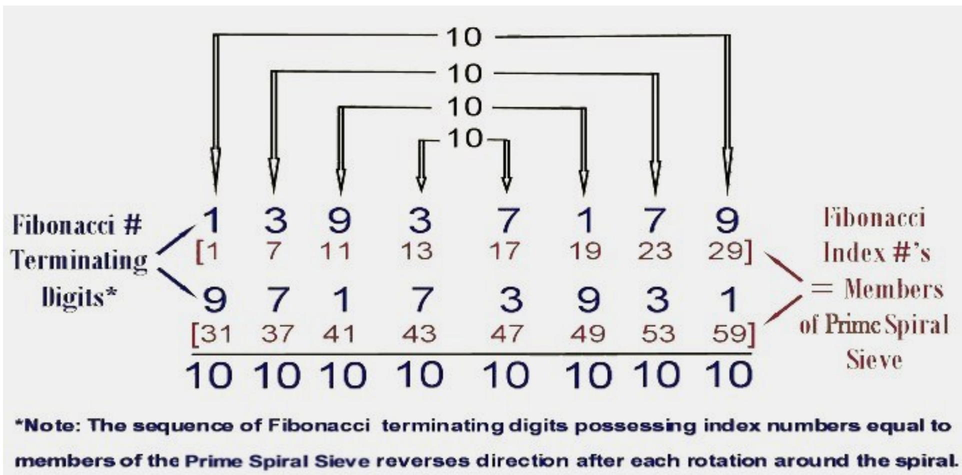
Given that all prime numbers > 5 are in the array, it is conjectured that this expression can be configured to generate all primes >5. What is critical to understand, is that the invisible hand of 2, 3 and 5, and their factorial 30, create the *structure* within which the balance of the prime numbers, i.e., all those greater than 5, are arrayed algorithmically–as we shall demonstrate. Primes 2, 3 and 5 play out in modulo 30-60-90 cycles (decomposing to {3,6,9} sequencing at the digital root level). Once the role of 2, 3 and 5 is properly understood, all else falls beautifully into place.

Deep Symmetries

The Prime Spiral Sieve possesses remarkable structural and numeric symmetries. For starters, the intervals between the prime roots (and every subsequent row or rotation of the sieve) are perfectly balanced, with a period 8 difference sequence of: {6, 4, 2, 4, 2, 4, 6, 2}. The entire domain can thus be defined as **1 {+6 +4 +2 +4 +2 +4 +6 +2} {repeat ... ∞}**. As we've already suggested, the number 30 figures large in our modulo 30 domain. The Prime Spiral Sieve is Archimedean in that the separation distance between turns equals 30, ad infinitum. The first two rotations increment as follows:



Interestingly, the sum of the 2nd rotation = 360. Is it coincidental that the product of the first three primorials, 2, 6 and 30 = 360? Or is it coincidental that when you multiply the first five [Fibonacci numbers](#) in sequence, you produce 1, 2, 6 and 30? And, speaking of the Fibonacci number sequence, there is symmetry mirroring the above in the relationship between the terminating digits of Fibonacci numbers and their index numbers equating to members of the array populating the Prime Spiral Sieve:



Remarkably, the sequence of Fibonacci terminating digits indexed to our domain (natural numbers not divisible by 2, 3 or 5), [13,937,179](#) (see graphic, above), is a prime number and a member of a twin prime pair (with 13,937,177). In case you're wondering, 13,937,179 is not a reversible prime (as the reversal is a semi-prime: 9,461 x 10,271 = 97,173,93). However, given all the repunits that follow, we take note that both of the reversal's factors are congruent to 11 (mod 30 & 90).

Perhaps most remarkable of all, 13,937,179 when added to its reversal 97,173,931 = 111,111,110 (in strict digital root terms, the sum is 11,111,111) and the entire repeating (and palindromic) Fibo sequence end-to-end (equivalent to two rotations around the sieve) gives you this palindromic equivalency: 1,393,717,997,173,931 ≡ 11,111,111 (mod 111,111,110)... (and interestingly, 11,111,111 * 111,111,110 = 1234567876543210 and 111,111,110/11,111,111 = 10). Also, 1,393,717,997,173,931 is divisible by the [repunits](#) 11 and 1,111 and 11,111,111.

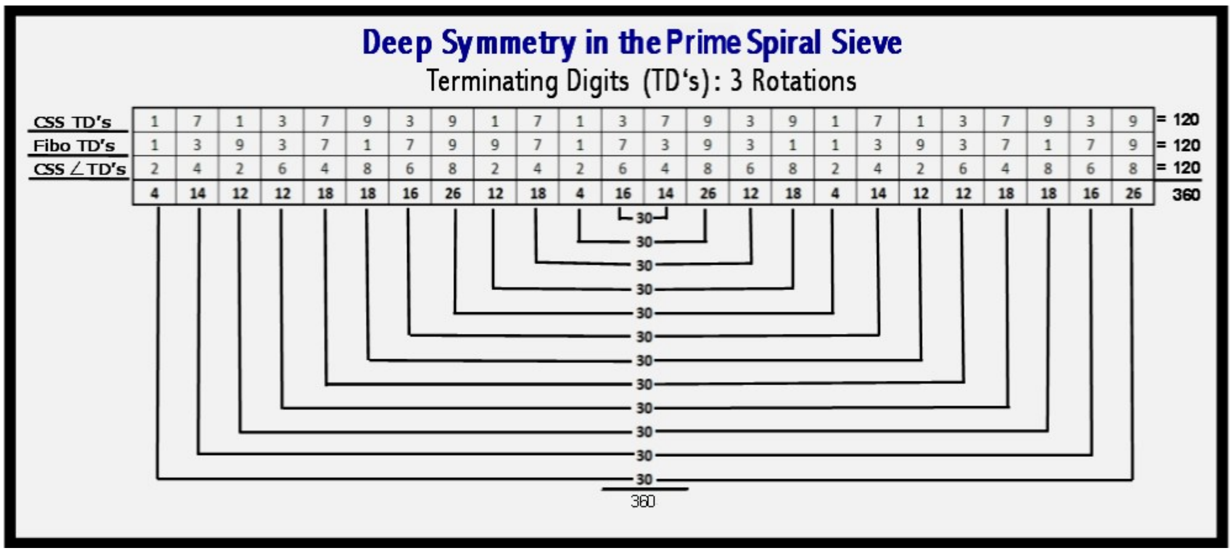
Another point of interest: the terminating digits of the first 8 Fibonacci numbers indexed to our domain (13937179) contain two each 1's, 3's, 7's, and 9's. This is also true of the terminating digits of the first eight members of our domain (17137939).

Echoing the Fibonacci patterns just described, the terminating digits of the *prime roots* (17,137,939), when added to *their* reversal (93,973,171) = 111,111,110. And, when you connect the prime root terminating digit sequence to its reversal, the entire palindromic sequence end-to-end produces this: 1,713,793,993,973,171 ≡ 111,111,111 (mod 111,111,110) [And in this case, 111,111,111 * 111,111,110 =

12345678876543210.]. And if that isn't enough, 1,713,793,993,973,171 is *also* divisible by the repunits 11 and 1,111 and 11,111,111.

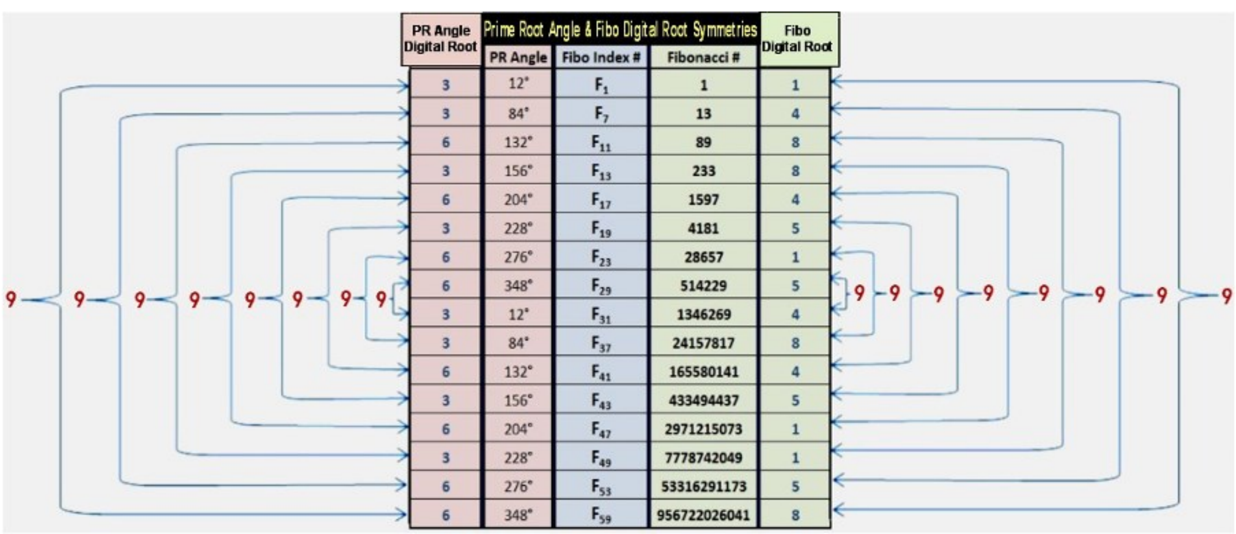
Well, not quite enough, because there's yet another related dimension of symmetry: The terminating digits of the prime root *angles* (24,264,868; see illustration of [Prime Spiral Sieve]) when added to *their* reversal (86,846,242) = 111,111,110, not to mention this sequence possesses symmetries that dovetail perfectly with the prime root and Fibo sequences, including the fact that when it is connected to its reversal (giving us 2,426,486,886,846,242), it's divisible by the repunits 11 and 1,111 and 11,111,111.

And when you combine the terminating digit symmetries described above, capturing three rotations around the sieve in their actual sequences, you produce the ultimate combinatorial symmetry:



CSS TD's	1	7	1	3	7	9	3	9	1	7	1	3	7	9	3	9	1	7	1	3	7	9	3	9	= 120
Fibo TD's	1	3	9	3	7	1	7	9	9	7	1	7	3	9	3	1	1	3	9	3	7	1	7	9	= 120
CSS ∠TD's	2	4	2	6	4	8	6	8	2	4	2	6	4	8	6	8	2	4	2	6	4	8	6	8	= 120
	4	14	12	12	18	18	16	26	12	18	4	16	14	26	12	18	4	14	12	12	18	18	16	26	360

Here's yet another fascinating dimension of symmetry: the pattern of 9's created by decomposing and summing either the digits of Fibonacci numbers indexed to the first two rotations of the spiral (a palindromic pattern {1393717997173931} that repeats every 16 Fibo index numbers) or, similarly, decomposing and summing the prime root angles. The decomposition works as follows (in digit sum arithmetic this would be termed summing to the digital root): F_{17} (the 17th Fibonacci number) = 1597 = 1 + 5 + 9 + 7 = 22 = 2 + 2 = 4:



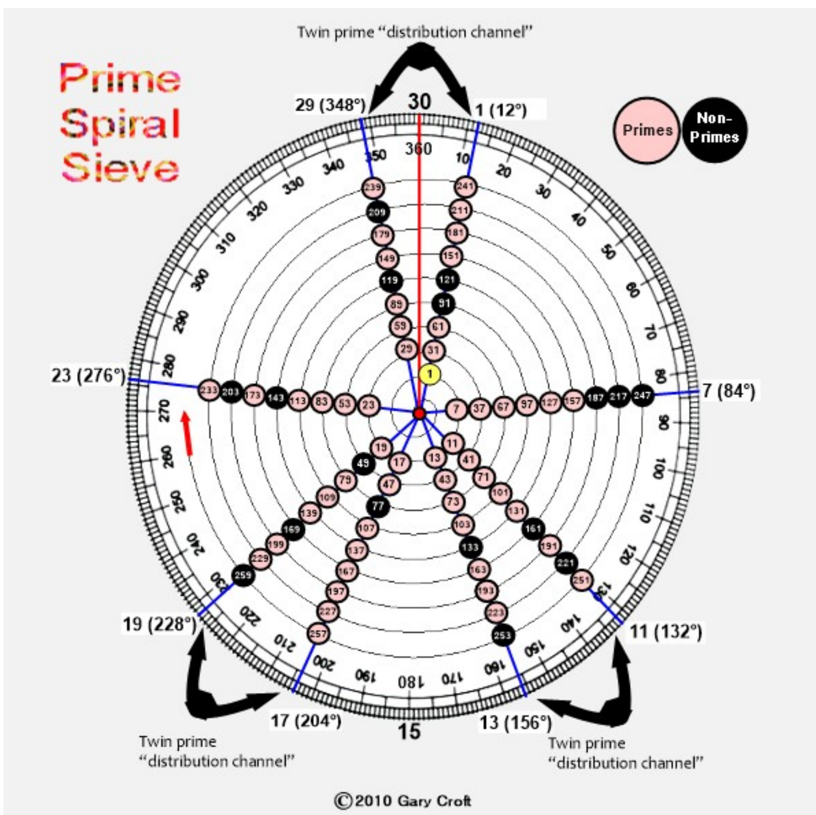
PR Angle Digital Root	PR Angle	Fibo Index #	Fibonacci #	Fibo Digital Root
3	12°	F_1	1	1
3	84°	F_7	13	4
6	132°	F_{11}	89	8
3	156°	F_{13}	233	8
6	204°	F_{17}	1597	4
3	228°	F_{19}	4181	5
6	276°	F_{23}	28657	1
6	348°	F_{29}	514229	5
3	12°	F_{31}	1346269	4
3	84°	F_{37}	24157817	8
6	132°	F_{41}	165580141	4
3	156°	F_{43}	433494437	5
6	204°	F_{47}	2971215073	1
3	228°	F_{49}	7778742049	1
6	276°	F_{53}	53316291173	5
6	348°	F_{59}	956722026041	8

Another dimension of symmetry involves the terminating digits of the prime roots and their angles: those paired with like terminating digits being separated by 120°: 1(12°) and 11(132°) ... 13(156°) and 23(276°) ... 7(84°) and 17(204°) ... 19(228°) and 29(348°). Another consideration with regard to terminating digits, is that one can easily construct, by combining all numbers with the same terminating digits, a four-fold arithmetic progression in increments of +10 and +20, starting with 1, 7, 13 and 19. Thus, combining 1(12°) and 11(132°) gives us: 1, 11, 31, 41, 61, 71, 91, [+10+20] ... n; combining 7(84°) and 17(204°) gives us 7, 17, 37, 47, 67, 77, 97, [+10+20] ... n; combining 13(156°) and 23(276°) gives us 13, 23, 43, 53, 73, 83, 103, [+10+20] ... n; and, combining 19(228°) and 29(348°) gives us 19, 29, 49, 59, 79, 89, 109, [+10+20] ...n. Looking at the array in this configuration, however, has borne no fruit.

As fascinating as the symmetries examined above may be, they are but a prelude to the beautiful patterns we'll explore when we discuss digital root sequencing and the Trinity of Triangles and Magic Squares rooted in Vedic Arithmetic that drive factorization algorithms within this domain. And, finally, if you want to jump ahead and view the most stunning symmetrical object found on this site, check out the Magic Mirror Matrix that maps factorizatons at the digital root level and accounts for the first 1000 prime numbers–exactly.

The Prime Spiral Sieve

Around the perimeter of the spiral sieve (pictured below) you'll note that the 8 radii are labeled in relation to their modulo 30 prime roots, i.e., 1(12°); 7(84°); 11(132°); 13(156°); 17(204°); 19(228°); 23(276°) and 29(348°). These relate to the fact that the circle is segmented into 30 equal sectors or radii separated by 12° (30*12°=360°), although only the eight radials that are the focus of this study are shown.



This sieve "exposes" the twin primes, aligning as they do along three distinct "distribution channels." One obvious implication, is that those numbers in the array congruent to {7} modulo 30 (radial angle 84°) and {23} modulo 30 (radial angle 276°) can be excluded as twin prime candidates (and, by definition, all prime numbers distributed along these two diagonals. with the exception of 7, which is twinned with 5, will be what are known as "isolated primes"). Later we explain how twin prime candidates can be segregated from all other positive integers and be partitioned into three columnar sets covertly aligned by the first three prime numbers (encoded in angles).

Conjectures and Facts Relating to the Prime Spiral Sieve

The array is rooted in the first three prime numbers: 2, 3 and 5 and their product, 30, the 3rd primorial. This array reveals that the first three primes play a very special role in creating the symmetrical geometries that align the distribution of all subsequent prime numbers, thus distinguishing them from all other primes. Primes 2, 3 and 5 are like 8-legged spiders assigned to spin the beautiful spiraling web in which the remaining prime numbers are arrayed along assigned threads. (For a detailed listing of Number 30's attributes, plus reference links click here: The Number 30).

It is conjectured that all (and only) the numbers in this array (and their negatives) can be derived using the interchangeable expression incorporating the first three prime numbers, 2, 3 and 5, where x=2, y=3 and z=5. Thus: $\mathbf{x^ny^n \pm z^n}$, $\mathbf{x^nz^n \pm y^n}$ and $\mathbf{y^nz^n \pm x^n}$. For example: 2 * 3 + 5 = 11 ... 2^3 * 5 - 3^3 = 13 ... 3^2 * 5 + 2 = 47 ... 5^2 * 3 - 2 = 73. To see more examples (1 thru 101) click here. This expression, therefore, potentially generates all numbers not divisible by its three terms, 2, 3 and 5, including all prime numbers >5. [Note: For any given number in the array, there are multiple–and possibly an infinite number of–solutions. For example, the number 11 can be expressed as xy+z = 11, x^2y^2-z^2 = 11, z^2y-x^6 = 11, etc.]

All prime numbers (with the exception of 2, 3 and 5) are distributed along 8 diagonals in intervals of 30, starting with "prime roots": 1, 7, 11, 13, 17, 19, 23 and 29 (thus: 1...31...61...91...n; 7...37...67...97...n; etc.).

The products of *any* combination of factors in the array = a number in the array, e.g., 7*11 = 77; 7*11*13 = 1001; etc. Conversely, all factors for composite numbers in the array can be found in the array.

Every composite number in our modulo 30 domain can be derived from the product of two terms in the domain multiplied together, and these multipliers need not necessarily be prime themselves. For example, 5831, which is congruent to 11, modulo 30, and therefore in the array, is the product of 49 x 119 = 5831. In this example, neither 49 (7 x 7) nor 119 (7 x 17) are prime, though both are members of the array.

The sum of *any* sequential odd number of addends in the array = a number in the array, e.g., 1+7+11 = 19; 1+7+11+13+17 = 49; etc.

Because the digital roots of all prime root angles are either 3 or 6, any prime root angle times another will produce a product whose digital root = 9, e.g., PR7 (84°) x PR29 (348°) = 84 x 348 = 29232 = dr(9).

Any number in the array x 30 + 1 = a number in the array.

The sum of the angles for 2(24°), 3(36°) and 5(60°) = 120°, and the sum of the prime roots (1+7+11+13+17+19+23+29) also = 120. This is because the prime roots are an arithmetic anagram for the angles of the first three primes, thus: 11+13 = 24; 17+19 = 36; and 1+7+23+29 = 60. The sum of the second rotation = 360 ... 3(2[24°] + 3[36°] + 5[60°]) = 30[360°]

The array reveals beautifully symmetrical relationships:

1[12°] + 29[348°] = 30[360°]

7[84°] + 23[276°] = 30[360°]
11[132°] + 19[228°] = 30[360°]
13[156°] + 17[204°] = 30[360°]

Mod 30 of all numbers in this array (and thus all primes other than 2, 3 and 5) must be 1, 7, 11, 13, 17, 19, 23 or 29.

The sum of the digital root sums of the prime roots (1, 7, 11, 13, 17, 19, 23, 29) = 1+7+2+4+8+1+5+2 = 30.

This sieve reveals why all primes >5 are adjacent to a multiple of six, as the prime root radii are adjacent to 6(72°); 12(144°); 18(216°); 24(288°); and 30(360°). [And you'll note that the digital root sums of all adjacent angles equal 9.]

The modulo 90 congruence of any member of this domain can be determined by digital root and terminating digit configured in an xy matrix: arrayed by digital root {1, 2, 4, 5, 7, 8} on the vertical axis and by terminating digits {1, 3, 7, 9} on the horizontal axis. Take, for example, number 179 (which happens to be prime): Its digital root = 8 and its terminating digit = 9. Looking at the table below, we can quickly determine that 179 is congruent to 89 modulo 90.

By definition prime factors for all composite numbers within this domain must originate from within it, which is why all composite numbers are reducible to one or more modulo 90 'congruence dyads' traceable to the first 24 members of this domain:

Determining Modulo 90 Congruence
(without division)
for Numbers ≡ to {1,7,11,13,17,19,23,29} mod 30
(aka numbers not divisible by 2, 3, or 5)
With Matrixed Digital Root ↔ Terminating Digit

Terminating Digit \ Digital Root	1	2	4	5	7	8
1	1	11	31	41	61	71
3	73	83	13	23	43	53
7	37	47	67	77	7	17
9	19	29	49	59	79	89

First 24 Terms:
n ≡ {1,7,11,13,17,19,23,29,31,37,41,43,47,49,53,59,61,67,71,73,77,79,83,89} mod 90
with period-24 digital root: {1,7,2,4,8,1,5,2,4,1,5,7,2,4,8,5,7,4,8,1,5,7,2,8} {repeat ...}
Note: *All* factor combinations for composite numbers within this domain can be rooted to a modulo 90 congruency dyad from the 24-member grid pictured above.

 10.03.2015

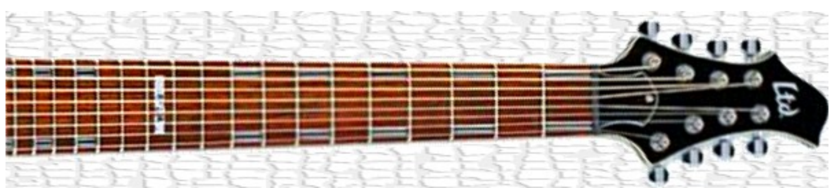
Factorization and Sieving Methods Employing the Prime Spiral Sieve

For a detailed discussion of efficient factorization and prime number sieving algorithms, as well as an in-depth analysis of the 8-chord progression and deterministic modulo 90 digital root dyad sequences underlying all factorizations employing this sieve, click here: **Prime Number Sieving Algorithms**.

If you use Python (the computer programming language designed for the development of scientific, engineering and mathematics applications) and want to cut to the chase, check out the MIT licensed Python module dubbed "pyprimes." It's designed to speed test both 'lazy' and 'fast and effective' *non-probabilistic* prime number sieving algorithms, including the Sieve of Eratosthenes, Wheel Factorization and the Prime Spiral Sieve (referred to by its alternative name, "Croft Spiral Sieve"), here or here at code.google.com. The programmer, we're pleased to report, rates the Prime Spiral Sieve "fastest/best" and recommends it as "the preferred way of generating prime numbers" compared to the several sieves tested.

Our use of the term 'factorization' requires an explanatory note: We're not referring to the 'top-down' prime factorization tree or "trial division" method you likely learned in high school. Rather, we're presenting a 'bottom-up' aproach using algorithms that deterministically project the locations within a spiral or on a matrix of all composite numbers and their not-necessarily-prime factors (meaning, every number populating the domain, save 1, is a 'factor' within the domain, regarldless of primality). Every composite number within the domain can be reduced to one or more factorization dyads. And, as we've aready alluded, all factors must be members of the domain.

Examining the Prime Spiral Sieve factorization sequences that index all composite numbers not divisible by 2, 3, or 5, we find multiplicative and additive-sum step algorithms that function in 'series-parallel.' There is zero computational waste, i.e., no trial-and-error used whatsoever. Every composite number in the domain is located with absolute precision–and with relatively high number-crunching avoidance, aka efficiency. There is no trial division. The algorithms in question generate period-8 structures (much like guitar chords on an 8-string guitar) that replicate clone-like along an 8 x n matrix (our virtual guitar's infinitely long fretboard).



As an example, here's a graphic showing the first two 8-note chords generated by 7 (the first sequence in a progression generated by numbers congruent to {7} modulo 30):

7-Chord Period-8 Patternization

	1	2	3	4	5	6	7	8	
n ≡ {	1	7	11	13	17	19	23	29	} mod 30
1	1	7	11	13	17	19	23	29	
2	31	37	41	43	47	**49**	53	59	
3	61	67	71	73	**77**	79	83	89	
4	**91**	97	101	103	107	109	113	**119**	
5	121	127	131	**133**	137	139	143	149	
6	151	157	**161**	163	167	169	173	179	
7	181	187	191	193	197	199	**203**	209	
8	211	**217**	221	223	227	229	233	239	
11	241	247	251	253	257	**259**	263	269	
12	271	277	281	283	**287**	289	293	299	
13	**301**	307	311	313	317	319	323	**329**	
14	331	337	341	**343**	347	349	353	359	
15	361	367	**371**	373	377	379	383	389	
16	391	397	401	403	407	409	**413**	419	
17	421	**427**	431	433	437	439	443	449	
	+30... ↓	+30... ↓	+30... ↓	+30... ↓	+30... ↓	+30... ↓	+30... ↓	+30... ↓	

+210 (rows 4–13, both sides)

7-Chord's period-8 algorithm indexes all members of the Factorization Domain divisible by (or a multiple of) 7. The *vertical* additive sum intervals separating notes = 7 x 30 = 210. 7-Chord expands geometrically in modulo 30 steps (viz. 7, 37, 67, 97, ...) with additive sum intervals of $30^2 = 900$.

And here we detail the calculations that generate the first four chords for numbers 7, 11, 13, 17, 19, 23, 29, 31, and 37 (the first number in our domain (1) being 'silent'):

Progressive Algorithms Account for All Composite Members of the Factorization Domain
Ultimately 'Leaving Behind' All Prime Numbers > 5

Factorization Domain: n ≡ { 1, 7, 11, 13, 17, 19, 23, 29 } modulo 30 (narrows sieving to 26.66...% of natural numbers)
Horizontal: Period-8 Sequential Multiplicative Muliples of n; Vertical: Cascading Additive Sums

n = 7		A	B	C	D	E	F	G	H	A/7	B/7	C/7	D/7	E/7	F/7	G/7	H/7
n^2 ... n x (n + ...) ...		7x7	7x11	7x13	7x17	7x19	7x23	7x29	7x31								
Period-8 Chord Notes		49	77	91	119	133	161	203	217	7	11	13	17	19	23	29	31
7 x 30 = 210	+	210	210	210	210	210	210	210	210								
Chord Note + Interval		259	287	301	329	343	371	413	427	37	41	43	47	49	53	59	61
	+	210	210	210	210	210	210	210	210								
Additive Sum		469	497	511	539	553	581	623	637	67	71	73	77	79	83	89	91
Progression (mod 30)	+	210	210	210	210	210	210	210	210								
= + 900		679	707	721	749	763	791	833	847	97	101	103	107	109	113	119	121
(see 37 x 30 = 1110)	+	210	210	210	210	210	210	210	210								

n = 11		A	B	C	D	E	F	G	H	A/11	B/11	C/11	D/11	E/11	F/11	G/11	H/11
		11x11	11x13	11x17	11x19	11x23	11x29	11x31	11x37								
		121	143	187	209	253	319	341	407	11	13	17	19	23	29	31	37
11 x 30 = 330	+	330	330	330	330	330	330	330	330								
		451	473	517	539	583	649	671	737	41	43	47	49	53	59	61	67
	+	330	330	330	330	330	330	330	330								
		781	803	847	869	913	979	1001	1067	71	73	77	79	83	89	91	97
	+	330	330	330	330	330	330	330	330								
		1111	1133	1177	1199	1243	1309	1331	1397	101	103	107	109	113	119	121	127
	+	330	330	330	330	330	330	330	330								

n = 13		A	B	C	D	E	F	G	H	A/13	B/13	C/13	D/13	E/13	F/13	G/13	H/13
		13x13	13x17	13x19	13x23	13x29	13x31	13x37	13x41								
		169	221	247	299	377	403	481	533	13	17	19	23	29	31	37	41
13 x 30 = 390	+	390	390	390	390	390	390	390	390								
		559	611	637	689	767	793	871	923	43	47	49	53	59	61	67	71
	+	390	390	390	390	390	390	390	390								
		949	1001	1027	1079	1157	1183	1261	1313	73	77	79	83	89	91	97	101
	+	390	390	390	390	390	390	390	390								
		1339	1391	1417	1469	1547	1573	1651	1703	103	107	109	113	119	121	127	131
	+	390	390	390	390	390	390	390	390								

n = 17		A	B	C	D	E	F	G	H	A/17	B/17	C/17	D/17	E/17	F/17	G/17	H/17
		17x17	17x19	17x23	17x29	17x31	17x37	17x41	17x43								
		289	323	391	493	527	629	697	731	17	19	23	29	31	37	41	43
17 x 30 = 510	+	510	510	510	510	510	510	510	510								
		799	833	901	1003	1037	1139	1207	1241	47	49	53	59	61	67	71	73
	+	510	510	510	510	510	510	510	510								
		1309	1343	1411	1513	1547	1649	1717	1751	77	79	83	89	91	97	101	103
	+	510	510	510	510	510	510	510	510								
		1819	1853	1921	2023	2057	2159	2227	2261	107	109	113	119	121	127	131	133
	+	510	510	510	510	510	510	510	510								

n = 19		A	B	C	D	E	F	G	H	A/19	B/19	C/19	D/19	E/19	F/19	G/17	H/19
		19x19	19x23	19x29	19x31	19x37	19x41	19x43	19x47								
		361	437	551	589	703	779	817	893	19	23	29	31	37	41	43	47
19 x 30 = 570	+	570	570	570	570	570	570	570	570								
		931	1007	1121	1159	1273	1349	1387	1463	49	53	59	61	67	71	73	77
	+	570	570	570	570	570	570	570	570								
		1501	1577	1691	1729	1843	1919	1957	2033	79	83	89	91	97	101	103	107
	+	570	570	570	570	570	570	570	570								
		2071	2147	2261	2299	2413	2489	2527	2603	109	113	119	121	127	131	133	137
	+	570	570	570	570	570	570	570	570								

		A	B	C	D	E	F	G	H	A/23	B/23	C/23	D/23	E/23	F/23	G/23	H/23
n = 23		23x23	23x29	23x31	23x37	23x41	23x43	23x47	23x49								
		529	667	713	851	943	989	1081	1127	23	29	31	37	41	43	47	49
23 x 30 = 690	+	690	690	690	690	690	690	690	690								
		1219	1357	1403	1541	1633	1679	1771	1817	53	59	61	67	71	73	77	79
	+	690	690	690	690	690	690	690	690								
		1909	2047	2093	2231	2323	2369	2461	2507	83	89	91	97	101	103	107	109
	+	690	690	690	690	690	690	690	690								
		2599	2737	2783	2921	3013	3059	3151	3197	113	119	121	127	131	133	137	139
	+	690	690	690	690	690	690	690	690								

		A	B	C	D	E	F	G	H	A/29	B/29	C/29	D/29	E/29	F/29	G/29	H/29
n = 29		29x29	29x31	29x37	29x41	29x43	29x47	29x49	29x53								
		841	899	1073	1189	1247	1363	1421	1537	29	31	37	41	43	47	49	53
29 x 30 = 870	+	870	870	870	870	870	870	870	870								
		1711	1769	1943	2059	2117	2233	2291	2407	59	61	67	71	73	77	79	83
	+	870	870	870	870	870	870	870	870								
		2581	2639	2813	2929	2987	3103	3161	3277	89	91	97	101	103	107	109	113
	+	870	870	870	870	870	870	870	870								
		3451	3509	3683	3799	3857	3973	4031	4147	119	121	127	131	133	137	139	143
	+	870	870	870	870	870	870	870	870								

		A	B	C	D	E	F	G	H	A/31	B/31	C/31	D/31	E/31	F/31	G/31	H/31
n = 31		31x31	31x37	31x41	31x43	31x47	31x49	31x53	31x59								
		961	1147	1271	1333	1457	1519	1643	1829	31	37	41	43	47	49	53	59
31 x 30 = 930	+	930	930	930	930	930	930	930	930								
		1891	2077	2201	2263	2387	2449	2573	2759	61	67	71	73	77	79	83	89
	+	930	930	930	930	930	930	930	930								
		2821	3007	3131	3193	3317	3379	3503	3689	91	97	101	103	107	109	113	119
	+	930	930	930	930	930	930	930	930								
		3751	3937	4061	4123	4247	4309	4433	4619	121	127	131	133	137	139	143	149
	+	930	930	930	930	930	930	930	930								

		A	B	C	D	E	F	G	H	A/37	B/37	C/37	D/37	E/37	F/37	G/37	H/37
n = 37																	
7 x 30 = 210		37x37	37x41	37x43	37x47	37x49	37x53	37x59	37x61								
210 + 900 = 1110		1369	1517	1591	1739	1813	1961	2183	2257	37	41	43	47	49	53	59	61
37 x 30 = 1110	+	1110	1110	1110	1110	1110	1110	1110	1110								
1110 + 900 = 2010		2479	2627	2701	2849	2923	3071	3293	3367	67	71	73	77	79	83	89	91
67 x 30 = 2010	+	1110	1110	1110	1110	1110	1110	1110	1110								
		3589	3737	3811	3959	4033	4181	4403	4477	97	101	103	107	109	113	119	121
	+	1110	1110	1110	1110	1110	1110	1110	1110								
		4699	4847	4921	5069	5143	5291	5513	5587	127	131	133	137	139	143	149	151
	+	1110	1110	1110	1110	1110	1110	1110	1110								

Before moving on from factorization, we must note that the power of the above described period-8, modulo 30 algorithms is leveraged significantly when their frame is tripled to create period-24 chords, modulo 90 (which expands the formula for intervals from n x 30 to n x 90). Given that our domain has a period-24 digital root, we can then parse the results by digital root. To illustrate, the graphic below shows the first 11-Chord, framed period-24, modulo 90. It would be an easy matter to 'hide' columns and sieve by any combination one would choose. For example, knowing that all twin prime candidates starting with (11, 13) must have digital root sequencing of either {2,4}, {5,7}, or {8,1}, we can easily determine the impact of 11-Chord sequencing on any combination of twin prime digital root variations we choose.

'11-chord' | Period-24 Factorization Sequence

Digital Root	1	7	2	4	8	1	5	2	4	1	5	7	2	4	8	5	7	4	8	1	5	7	2	8	{repea
n ≡ {	1	7	11	13	17	19	23	29	31	37	41	43	47	49	53	59	61	67	71	73	77	79	83	89	} mo
	1	7	11	13	17	19	23	29	31	37	41	43	47	49	53	59	61	67	71	73	77	79	83	89	
↑	91	97	101	103	107	109	113	119	11x11	127	131	133	137	139	11x13	149	151	157	161	163	167	169	173	179	
	181	11x17	191	193	197	199	133	11x19	211	217	221	223	227	229	233	239	241	247	251	11x23	257	259	263	269	
	271	277	281	283	287	289	293	299	301	307	311	313	317	11x29	323	329	331	337	11x31	343	347	349	353	359	
	361	367	371	373	377	379	383	389	391	397	401	403	11x37	409	413	419	421	427	431	433	437	439	443	449	
↑	11x41	457	461	463	467	469	11x43	479	481	487	491	493	497	499	503	509	511	11x47	521	523	527	529	533	11x49	
	541	547	551	553	557	559	563	569	571	577	581	11x53	587	589	593	599	601	607	611	613	617	619	623	629	
	631	637	641	643	647	11x59	653	659	661	667	671	673	677	679	683	689	691	697	701	703	707	709	713	719	
	721	727	731	733	11x67	739	743	749	751	757	761	763	767	769	773	779	11x71	787	791	793	797	799	11x73	809	
	811	817	821	823	827	829	833	839	841	11x77	851	853	857	859	863	11x79	871	877	881	883	887	889	893	899	
	901	907	911	11x83	917	919	923	929	931	937	941	943	947	949	953	959	961	967	971	973	977	11x89	983	989	
↓	991	997	11x91	1003	1007	1009	1013	1019	1021	1027	1031	1033	1037	1039	1043	1049	1051	1057	1061	1063	11x97	1069	1073	1079	
	1081	1087	1091	1093	1097	1099	1103	1109	11x101	1117	1121	1123	1127	1129	1133	1139	1141	1147	1151	1153	1157	1159	1163	1169	
	1171	1177	1181	1183	1187	1189	1193	1199	1201	1207	1211	1213	1217	1219	1223	1229	1231	1237	1241	1243	1247	1249	1253	1259	
	1261	1267	1271	1273	1277	1279	1283	1289	1291	1297	1301	1303	1307	1309	1313	1319	1321	1327	1331	1333	1337	1339	1343	1349	
	1351	1357	1361	1363	1367	1369	1373	1379	1381	1387	1391	1393	1397	1399	1403	1409	1411	1417	1421	1423	1427	1429	1433	1439	
↓	1441	1447	1451	1453	1457	1459	1463	1469	1471	1477	1481	1483	1487	1489	1493	1499	1501	1507	1511	1513	1517	1519	1523	1529	

11-Chords Span 11 Rows

'Note' Intervals Span 11 Rows

Gary W. Croft

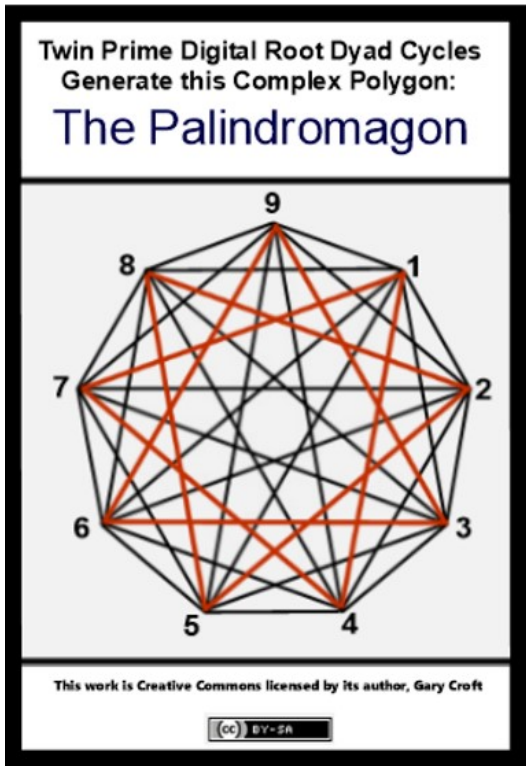
Counting Primes

The Prime Spiral Sieve can be used to calculate the number of prime numbers within a stated range (from 0 to n, or π(x) using the standard notation) with absolute precision, albeit in several unconventional steps. It does so employing the four elementary operations of arithmetic: addition, subtraction, multiplication and division, and the taking of one square root. You could say that this approach is more recipe than function, algorithm, or asymtotic analysis. Regardless, what makes this method novel is that it parses and gives the number of primes *without the need to identify any specific primes other than the first three*: **2, 3,** and **5**, i.e., there's no requirement for primality testing, per se. For an in depth exploration of this methodology, click here: **Prime Number Counting Methodology**

Twin Primes

The world of twin primes is truly magical. For a detailed discussion of the factorization algorithms and symmetry groups (permutating matrices, orthogonal Latin squares, palindromes, equilateral triangles, complex polygons, etc.) that ultimately determine the distribution of twin primes along three modulo 30 'channels' go to this page: **Twin Primes Demystified: Distribution Algorithms and Symmetries**.

There you will be introduced to the beautiful 'Palindromagon' (pictured below), a complex polygon generated by tiered digital root dyad cycles central to the twin prime distribution channels (as well as modulo 90 factorization sequences). Its name comes from the fact that the triangulations generating it sum to a period-18 palindrome consisting of the six possible permutations of {3,6,9}, which in turn can be permutated to produce two 3 x 3 Latin squares with rows, columns and principal diagonals all summing to 18:



Distribution of Perfect Squares

All perfect squares within our domain (numbers not divisible by 2, 3 or 5) possess a digital root of 1, 4 or 7 and are congruent to either {1} or {19} modulo 30. By definition, this includes the squares of all prime numbers greater than 5. We can easily explain this from a digital root perspective given that the digital roots of members of our domain are restricted to 1, 2, 4, 5, 7 or 8 (Numbers with digital root 3, 6, or 9 can't be members because they are divisible by 3.). Thus the digital root of squares is likewise restricted, as follows (and note the palindrome):

1 x 1 = 1
2 x 2 = 4
4 x 4 = 7
5 x 5 = 7
7 x 7 = 4
8 x 8 = 1

By arithmetic law, perfect squares can only have terminating digits of 1, 4, 5, 6, 9 or 0. Only two of these final digits (1 and 9) apply to our domain, i.e., for numbers congruent to {1, 11, 19 or 29} modulo 30. In turn, numbers congruent to {11, 29} sequence digital roots 2, 5 or 8, and therefore – as we demonstrated above – there can be no perfect squares among them. And so it is that the distribution of squares is narrowed to numbers congruent to {1, 19} modulo 30, which is to say they distribute along two – and only two – radii of the Prime Spiral Sieve: 12° (numbers congruent to {1} modulo 30) and 228° (numbers congruent to {19} modulo 30). (This is also consistent with the fact that the quadratic residues for modulo 30 (making them congruent with perfect squares) are 1 and 19.)

[And it follows that all squares in this series distribute evenly to two of the three twin prime distribution channels, described above, negating a significant percentage of potential twin prime pairs.]

The matrix below illustrates the distribution of squares from 1*1 thru 59*59 (squares hi-lited in blue):

12°	84°	132°	156°	204°	228°	276°	348°
1	7	11	13	17	19	23	29
A	B	C	D	E	F	G	H
AA (1*1)	AB (1*7)	AC (1*11)	AD (1*13)	AE (1*17)	AF (1*19)	AG (1*23)	AH (1*29)
BD (7*13)	BA (7*31)	BG (7*23)	BF (7*19)	BC (7*11)	BB (7*7)	BH (7*29)	BE (7*17)
CC (11*11)	CE (11*17)	CA (11*31)	CG (11*13)	CB (11*37)	CH (11*29)	CD (11*13)	CF (11*19)
DB (13*37)	DF (13*19)	DE (13*17)	DA (13*31)	DH (13*29)	DD (13*13)	DC (13*41)	DG (13*23)
EG (17*23)	EC (17*41)	ED (17*43)	EH (17*29)	EA (17*31)	EE (17*17)	EF (17*19)	EB (17*37)
FF (19*19)	FD (19*43)	FH (19*29)	FB (19*37)	FG (19*23)	FA (19*31)	FE (19*47)	FC (19*41)
GE (23*47)	GH (23*29)	GB (23*37)	GC (23*41)	GF (23*49)	GG (23*23)	GA (23*31)	GD (23*43)
HH (29*29)	HG (29*53)	HF (29*49)	HE (29*47)	HD (29*43)	HC (29*41)	HB (29*37)	HA (29*31)
12°	84°	132°	156°	204°	228°	276°	348°
1	7	11	13	17	19	23	29
A	B	C	D	E	F	G	H
AA (31*31)	AB (31*37)	AC (31*41)	AD (31*43)	AE (31*47)	AF (31*49)	AG (31*53)	AH (31*59)
BD (37*43)	BA (37*61)	BG (37*53)	BF (37*49)	BC (37*41)	BB (37*37)	BH (37*59)	BE (37*47)
CC (41*41)	CE (41*47)	CA (41*61)	CG (41*53)	CB (41*67)	CH (41*59)	CD (41*43)	CF (41*49)
DB (43*67)	DF (43*49)	DE (43*47)	DA (43*61)	DH (43*59)	DD (43*43)	DC (43*71)	DG (43*53)
EG (47*53)	EC (47*71)	ED (47*73)	EH (47*59)	EA (47*61)	EE (47*47)	EF (47*49)	EB (47*67)
FF (49*49)	FD (49*73)	FH (49*59)	FB (49*67)	FG (49*53)	FA (49*61)	FE (49*77)	FC (49*71)
GE (53*77)	GH (53*59)	GB (53*67)	GC (53*71)	GF (53*79)	GG (53*53)	GA (53*61)	GD (53*73)
HH (59*59)	HG (59*83)	HF (59*79)	HE (59*77)	HD (59*73)	HC (59*71)	HB (59*67)	HA (59*61)

Summarizing the above relationships in mathematical terms (and in the knowledge that these modular relationships apply to the squares of all prime numbers ≥ 7) we get:

for all n where n mod 30 = 1, n^2 mod 30 = 1

for all n where n mod 30 = 29, n^2 mod 30 = 1

for all n where n mod 30 = 11, n^2 mod 30 = 1

for all n where n mod 30 = 19, n^2 mod 30 = 1

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for all n where n mod 30 = 7, n^2 mod 30 = 19

for all n where n mod 30 = 23, n^2 mod 30 = 19

for all n where n mod 30 = 13, n^2 mod 30 = 19

for all n where n mod 30 = 17, n^2 mod 30 = 19

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When the digital root of perfect squares is sequenced within a modulo 30 x 3 = modulo 90 horizon, beautiful symmetries in the form of 24 period repeating palindromes are revealed, which the author has documented on the *On-Line Encyclopedia of Integer Sequences* as [Digital root of squares of numbers not divisible by 2, 3 or 5 (A24092)]:

1, 4, 4, 7, 1, 1, 7, 4, 7, 1, 7, 4, 4, 7, 1, 7, 4, 7, 1, 1, 7, 4, 4, 1

In the matrix pictured below, we list the first 24 elements of our domain, take their squares, calculate the modulo 90 congruence and digital roots of each square, and display the digital root factorization dyad for each square (and map their collective bilateral 9 sum symmetry):

Digital Root of Perfect Squares

Congruent to {1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53 59, 61, 67, 71, 73, 77, 79, 83, 89} modulo 90

⭑ A Period 24 Repeating Palindromic Sequence ⭑

Index	PRS*	PRS^2	PRS^2 mod 90	PRS^2 digital root	**	PRS^2 digital root factors***
1	**1**	1	1	1	A	1 x 1
2	**7**	49	49	4	E	7 x 7
3	**11**	121	31	4	B	2 x 2
4	**13**	169	79	7	C	4 x 4
5	**17**	289	19	1	F	8 x 8
6	**19**	361	1	1	A	1 x 1
7	**23**	529	79	7	D	5 x 5
8	**29**	841	31	4	B	2 x 2
9	**31**	961	61	7	C	4 x 4
10	**37**	1369	19	1	A	1 x 1
11	**41**	1681	61	7	D	5 x 5
12	**43**	1849	49	4	E	7 x 7
13	**47**	2209	49	4	B	2 x 2
14	**49**	2401	61	7	C	4 x 4
15	**53**	2809	19	1	F	8 x 8
16	**59**	3481	61	7	D	5 x 5
17	**61**	3721	31	4	E	7 x 7
18	**67**	4489	79	7	C	4 x 4
19	**71**	5041	1	1	F	8 x 8
20	**73**	5329	19	1	A	1 x 1
21	**77**	5929	79	7	D	5 x 5
22	**79**	6241	31	4	E	7 x 7
23	**83**	6889	49	4	B	2 x 2
24	**89**	7921	1	1	F	8 x 8

↕ bilateral 9 sums ↕

↑ {palindrome} ↑
↓ {repeat} ↓

↓ {repeat} ↓

* PRS: Prime Root Set = Natural numbers not divisible by 2, 3 or 5
** See Tables, Below *** See Tables, Below

A:	1 x 1 = 1	A:	≡ {1 or 19} moduo 90	← digital root = 1
B:	2 x 2 = 4	B:	≡ {31 or 49} moduo 90	← digital root = 4
C:	4 x 4 = 7	C:	≡ {61 or 79} moduo 90	← digital root = 7
D:	5 x 5 = 7	D:	≡ {61 or 79} moduo 90	← digital root = 7
E:	7 x 7 = 4	E:	≡ {31 or 49} moduo 90	← digital root = 4
F:	8 x 8 = 1	F:	≡ {1 or 19} moduo 90	← digital root = 1

Here's one more modulo 90 spin on perfect squares. Parsing the squares by their mod 90 congruence reveals that there are 96 perfect squares generated with each 4 * 90 = 360 degree cycle, which distribute 16 squares to each of 6 mod 90 congruence sub-sets defined as n congruent to {1, 19, 31, 49, 61, 79} forming 4 bilateral 80 sums. Note that each of the 6 columns has 8 bilateral 360 sums, tor a total of 48 * 360 = 40 * 432 (much more on the significance of number 432, elsewhere on this site). Here's the graphic:

Modulo 90 & Digital Root (dr) of (n ≡ {1, 7, 11, 13, 17, 19, 23, 29} mod 30)^2

(Which includes the squares of all prime numbers > 5)

Distribute Evenly to n ≡ {1, 19, 31, 49, 61, 79} mod 90 & Digital Roots 1, 4 & 7*

	n^2 dr	1	1	4	4	7	7	
	$n^2 \equiv \{$	1	19	31	49	61	79	} mod 90
1	n	1	17	11	7	31	13	
	n^2	1	289	121	49	961	169	
2	n	19	37	29	43	41	23	
	n^2	361	1369	841	1849	1681	529	
3	n	71	53	61	47	49	67	
	n^2	5041	2809	3721	2209	2401	4489	
4	n	89	73	79	83	59	77	
	n^2	7921	5329	6241	6889	3481	5929	
5	n	91	107	101	97	121	103	
	n^2	8281	11449	10201	9409	14641	10609	
6	n	109	127	119	133	131	113	
	n^2	11881	16129	14161	17689	17161	12769	
7	n	161	143	151	137	139	157	
	n^2	25921	20449	22801	18769	19321	24649	
8	n	179	163	169	173	149	167	
	n^2	32041	26569	28561	29929	22201	27889	
9	n	181	197	191	187	211	193	
	n^2	32761	38809	36481	34969	44521	37249	
10	n	199	217	209	223	221	203	
	n^2	39601	47089	43681	49729	48841	41209	
11	n	251	233	241	227	229	247	
	n^2	63001	54289	58081	51529	52441	61009	
12	n	269	253	259	263	239	257	
	n^2	72361	64009	67081	69169	57121	66049	
13	n	271	287	281	277	301	283	
	n^2	73441	82369	78961	76729	90601	80089	
14	n	289	307	299	313	311	293	
	n^2	83521	94249	89401	97969	96721	85849	
15	n	341	323	331	317	319	337	
	n^2	116281	104329	109561	100489	101761	113569	
16	n	359	343	349	353	329	347	
	n^2	128881	117649	121801	124609	108241	120409	

* Table covers $1^2 \rightarrow 359^2$ where 4 mod 90 cycles traverses 360°.
There are 6 * 16 = 96 perfect squares in each 360 degree cycle.
Each of the 6 columns has 8 bilateral 360 sums: 48 * 360 = 40 * 432.

Before leaving the subject of perfect squares, there's another hidden dimension of our domain worth noting involving multiples of 360, i.e., when framed as n ≡ {1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53 59, 61, 67, 71, 73, 77, 79, 83, 89} modulo 90, and taking 'bipolar' differentials of perfect squares, as follows:

$(89^2 - 1^2) = 360 \times 22$

$(83^2 - 7^2) = 360 \times 19$

$(79^2 - 11^2) = 360 \times 17$

$(77^2 - 13^2) = 360 \times 16$

$(73^2 - 17^2) = 360 \times 14$

$(71^2 - 19^2) = 360 \times 13$

$(67^2 - 23^2) = 360 \times 11$

$(61^2 - 29^2) = 360 \times 8$

$(59^2 - 31^2) = 360 \times 7$

$(53^2 - 37^2) = 360 \times 4$

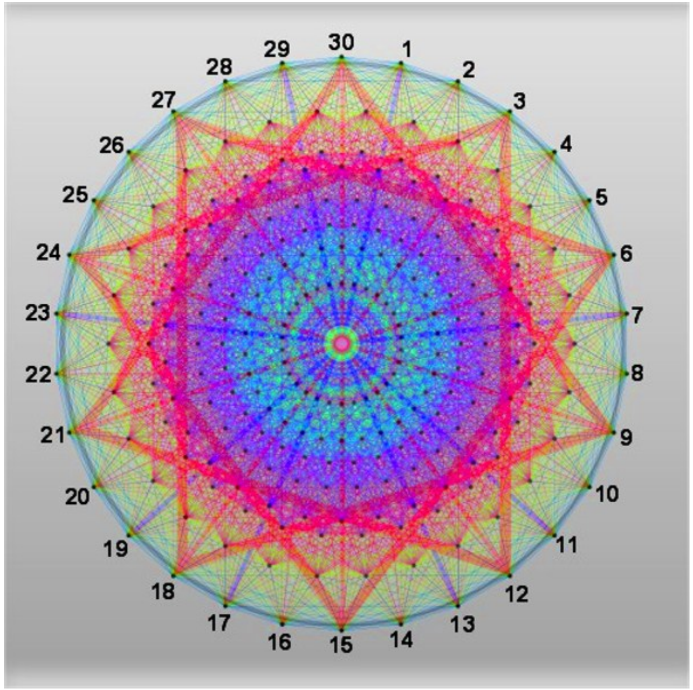
$(49^2 - 41^2) = 360 \times 2$

$(47^2 - 43^2) = 360 \times 1$

From Alpha to Omega

The Ulam Spiral arrays prime numbers in fragmented spiral and diagonal formations. Quoting from Wikipedia: "Since in the Ulam spiral adjacent diagonals are alternatively odd and even numbers, it is no surprise that all prime numbers lie in alternate diagonals ... What is startling is the tendency of prime numbers to lie on some diagonals more than others." From this one might deduce that the Ulam Spiral is very likely a scrambled version of the Prime Spiral Sieve as the latter demonstrates how all prime numbers (except 2, 3 and 5) are fundamentally arrayed along eight (and only eight) diagonals.

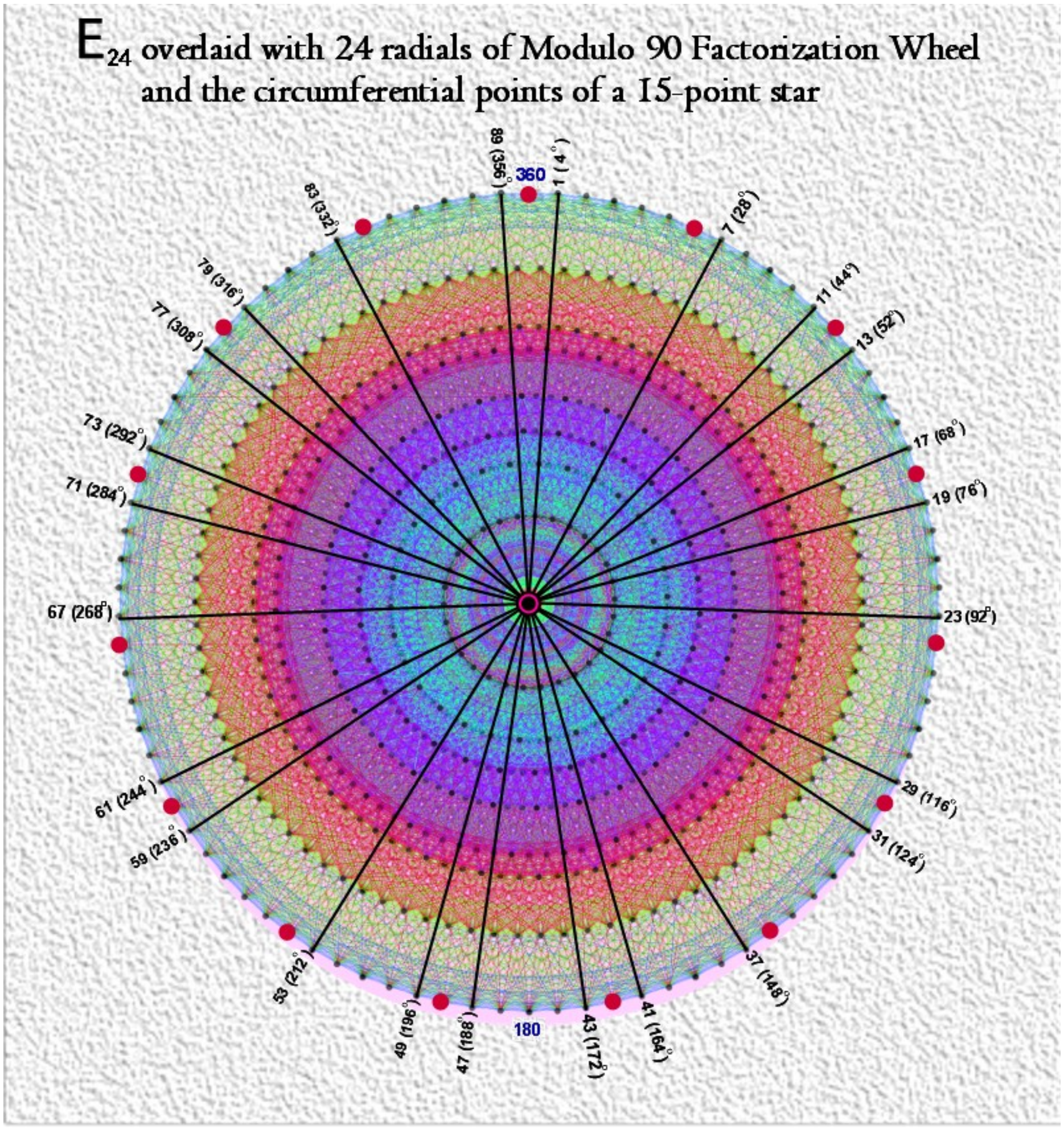
It would appear, circumstantially, that the Prime Spiral Sieve is mathematically harmonious and perhaps isomorphic with the most complex and visually arresting Lie group, named E_8, which–like the Prime Spiral Sieve–is 8-dimensional (E_8 is pictured below superimposed with a star polygon and the 8 radii of the modulo 30 factorization wheel). This group was recently in the news as possibly being a key to unifying theories in gravity and particle physics to create the proverbial "theory of everything." The number 30–integral to the Prime Spiral Sieve–is the Coxeter Group number *h*, dual Coxeter number and the highest degree of fundamental invariance of E_8. You'll note, looking at the graphical representation of E_8 below, that the perimeters of every one of its multiple concentric circles possesses 30 points. And, not surprisingly, E_8 has 2-, 3- and 5-torsion and its exponents are the co-primes up to 30, i.e., 1, 7, 11, 13, 17, 19, 23, and 29–numbers you're very familiar with if you've read to this point ... which brings us full circle O:



The Number 30 in Lie Group E_8

This graphic superimposes images of the star polygon, modulo 30 wheel factorization radii and "E_8 graph of the Gosset 421 polytope as a 2-dimensional skew orthogonal projection inside Petrie polygon ... an emulation of the hand drawn original by Peter McMullen" licensed by Creative Commons; license terms here. (For 3-D version of this graphic step back three feet from the screen.)

We'll close with a graphic showing E_{24} superimposed with the 24 radials of a modulo 90 factorization wheel and the 15 points of a 15-point star represented with red dots, each point separated by 24° (and we see that *all nine* twin prime distribution channels are hit dead center).



Your feedback welcome! Email: gwc@hemiboso.com

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